Method of Lines with Special Absorbing Boundary Conditions—Analysis of Weakly Guiding Optical Structures

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Abstract—**It is shown that the introduction of special absorbing boundary conditions (SABC) in the method of lines (MoL) for applications in the integrated optics allows to make the discretized area smaller. The numerical effort decreases substantially. This effect will be demonstrated in case of a slab waveguide.**

I. Introduction

EXACT models for the waveguides in the integrated optics must be provided for the simulation of optical devices. Quasiplanar waveguides can be analyzed by several numerical methods, e.g., the finite difference method [1], the finite-element method [2] and the method of lines (MoL) [3]. It was demonstrated in [4], [5] that the MoL is very suitable for the analysis of optical waveguides.

The MoL is a semianalytical method, i.e., the wave equations are discretized only as far as necessary and in one direction the calculation is done analytically, which yields accurate results with less computational effort. Round-off errors can be neglected. The MoL behaves stationary [6] and therefore, the convergence curves are monotonic [4] and an extrapolation to the accurate results is possible. The MoL stands in relation to the discrete Fourier transformation and therefore the calculation of the field components on the discretization lines is very accurate. Discontinuous field curves can be described accurately because the interface conditions are included in the method [4]. Structures with extreme large permittivity differences, like those which appear at the transition from dielectric to metal with finite conductivity can also be analyzed with the MoL by including complex or imaginary permittivities [7], [8]. A further advantage of the MoL is the relatively easy and clear formulation of the discretization by introduction of difference operators.

In the MoL and in other methods, the structure has to be surrounded by a box of metallic or magnetic walls, whereby box modes can occur and complicate the numerical evaluation. Because these boundaries must not have any influence on the field distribution, the distance between the walls must be chosen very large, especially in case of a structure with a weak confinement. For such a structure the numerical effort is high.

It was demonstrated in [9] that these disadvantages can be vanquished by introduction of absorbing boundary conditions (ABC) in the analysis of microstrip waveguides. Also for the analysis of longitudinal variant waveguides the ABC were used very successful [10].

In this letter, special absorbing boundary conditions (SABC) are introduced for the calculation of the eigenmodes of optical waveguides (see also [11]) and the advantages in relation to the hitherto used Dirichlet and Neumann boundary conditions will be elucidated.

II. Theory

For the discretization of the Helmholtz equation and the Sturm–Liouville equation, the difference operator $\boldsymbol{D}$ was introduced taking into account the Dirichlet and Neumann boundary condition. In case of the Dirichlet condition, we obtain with $\psi_0 = 0$ and $\psi_{N+1} = 0$:

$$\begin{array}{cc} & \psi_0\ [\psi_1, \psi_2, \cdots, \psi_{N-1}, \psi_N]\ \psi_{N+1} \\ h\dfrac{\partial \psi}{\partial x} \rightarrow & \begin{matrix} -1 \\ \\ \\ \\ \\ \\ \end{matrix} \underbrace{\begin{bmatrix} 1 & & & & \\ -1 & 1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \\ & & & & -1 \end{bmatrix}}_{\boldsymbol{D}} \begin{matrix} \\ \\ \\ \\ \\ 1 \end{matrix} \end{array}$$

By factorization [12] of the wave equation and under utilization of the fact, that the waves are supposed to fulfill the Sommerfeld's radiation condition [13] at the left and right boundary, the potentials ψ_0 and ψ_{N+1} on the boundary are expressed through potentials inside the computational window. We obtain the following expressions

$$\psi_0 = -a_l\psi_1 + b_l\psi_2$$
$$\psi_{N+1} = b_r\psi_{N-1} - a_r\psi_N$$

with

$$a = -\frac{2 - n_r^2}{1 + n_r}, \qquad b = -\frac{1 - n_r}{1 + n_r}$$

and

$$n_r = \overline{h}\sqrt{\varepsilon_{re} - \varepsilon_r} \qquad \varepsilon_{re} = \frac{k_z^2}{k_0^2} = n_{\text{eff}}^2 \qquad \overline{h} = k_0 h,$$

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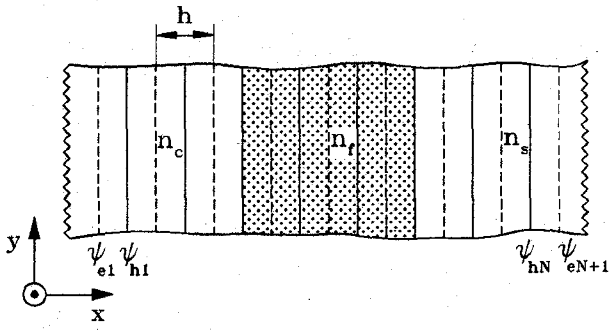

Fig. 1. Slab waveguide with $n_s = 3.40$, $n_f = 3.44$, $n_c = 1.0$ and wavelength $\lambda = 1.55\mu$m.

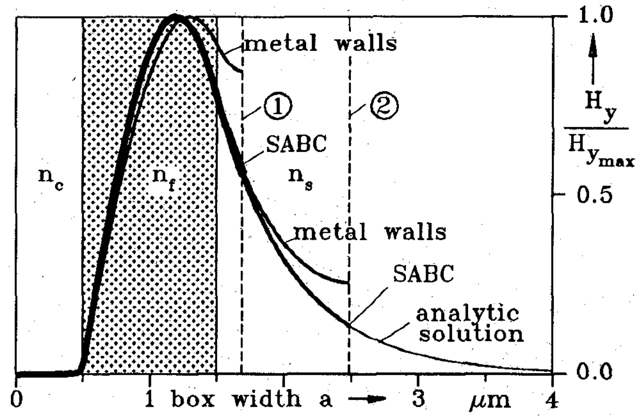

Fig. 2. Normalized y component of the magnetic field for SABC and metallic walls at the positions 1 and 2.

which can be included in a new difference operator $\boldsymbol{D}_a$. The difference operator $\boldsymbol{D}_a$ is represented in the following way:

$$\boldsymbol{D}_a = \begin{bmatrix} (a_l+1) & -b_l & & & \\ -1 & 1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \\ & & & b_r & -(a_r+1) \end{bmatrix}.$$

The regions in which the SABC are positioned have a lower refractive index than the effective index. For that very reason the coefficients a and b are real. That means, that the SABC describes the decaying character of the field in an infinite halfspace beyond the boundary. With the new difference operator $\boldsymbol{D}_a$ the second derivative with respect to the x variable, which occurs in the Helmholtz equation is described as

$$\bar{h}^2 \frac{\partial^2 \psi_h}{\partial \bar{x}^2} \longrightarrow \underbrace{-\boldsymbol{D}^t \boldsymbol{D}_a}_{\boldsymbol{P}_h} \boldsymbol{\psi}_h \ ,$$

whereas the second derivative in the Sturm–Liouville equation is expressed in discretized form by the matrix product:

$$\bar{h}^2\, \epsilon_r(\bar{x}) \frac{\partial}{\partial \bar{x}} \left(\frac{1}{\epsilon_r(\bar{x})} \frac{\partial \psi_e}{\partial \bar{x}} \right) \longrightarrow \underbrace{-\epsilon_e \boldsymbol{D}_a \epsilon_h^{-1} \boldsymbol{D}^t}_{\boldsymbol{P}_e} \boldsymbol{\psi}_e$$

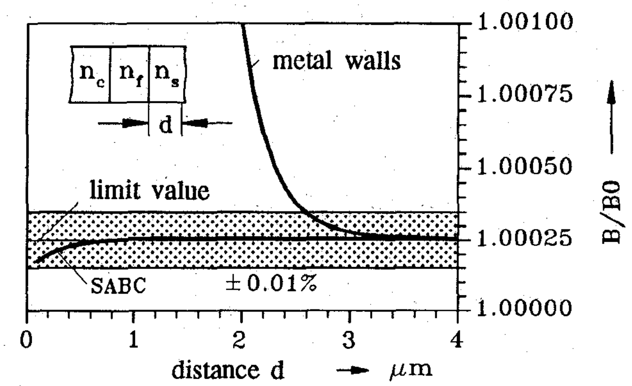

Fig. 3. Convergence behavior of the phase parameter B for TM-polarization with $B = (n_{\text{eff}}^2 - n_s^2)/(n_f^2 - n_s^2)$ and $B_0 = 0.38508056$.

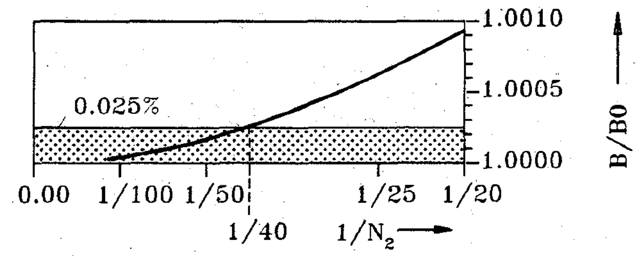

Fig. 4. Dependence of the limit value B from the number N_2 of the lines in the film.

The further analysis can be accomplished with these matrixes $\boldsymbol{P}_e$ and $\boldsymbol{P}_h$ like in [4].

III. Results

Now, we will exhibit the advantages of the introduction of the SABC opposed to the hitherto used metallic or magnetic walls for the example of the slab waveguide, which is shown in Fig. 1. Here we can demonstrate the influence of the SABC without disturbances from other effects. The slab waveguide is particularly suited as a reference structure, because we can compare our results with the well known analytical solution. Fig. 2 illustrates the field distributions of the slab waveguide for TM-polarization for metallic walls respectively SABC at the positions 1 and 2. The field curves elucidate the different influences of the metal wall and the SABC on the wave propagation in the slab waveguide. It is seen in Fig. 2 that for SABC already at the position 1, where the SABC is very close to the film, the field distribution agrees very good with the analytical solution, whereas for the metallic walls the disturbance is substantial at the position 1. Only for very far distances between the metallic wall and the film the deviation from the analytical solution will be smaller. Adequate strong disturbances of the field distributions will be obtained by using magnetic walls. The fact that the SABC can be positioned very near to the film is illustrated in Fig. 3, where the phase parameter B normalized by the analytical value B_0 is presented versus the distance d between the film and the wall. The limit value is reached in case of the SABC with an error of 0.01% for very small distances d, whereas in case of the metallic walls the limit value is reached only for big distances, that means that the discretized area must be chosen very large and for that very reason the corresponding matrices become huge and that raises the numerical effort. We see that the limit value is not identical with the analytical value. The reason for this is that we have a finite discretization. For an infinite discretization we would reach the analytical limit value. Fig. 4 shows clearly that for 40 lines in the film the deviation is only 0.025%. It is demonstrated in Fig. 3 and Fig. 4 that the SABC do not influence the stability of the eigenmode analysis by the MoL described in [4] and [6].

For the TE-polarization it is also possible to place the SABC very close to the film.

IV. Conclusion

The introduction of the special absorbing boundary conditions in the method of lines is very advantageous because the SABC can be placed closer to the guiding structure, whereby the discretized area and the corresponding matrices become smaller. With the SABC we simulate an infinite computational window at the place where they are introduced and box modes, which complicate the numerical evaluation, do not occur.

The inclusion of the SABC in the analysis of ridge waveguides will be treated in our future papers.

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